Mathematics 31 / 31 IB Formula Sheet

BASIC DIFFERENTIATION RULES

First Principles: If $y = f(x)$ then $\frac{dy}{dx}$ *or* $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Power Rule: If $y = x^n$ then $\frac{d}{dx}\left[x^n\right] = n\,x^{n-1}$

Constant Rule: If $y = k$ then $\frac{d[k]}{dx} = 0$

Product Rule: If $y = u\cdot v$ then $\frac{d}{dx}[uv] = u\cdot\frac{dv}{dx} + v\cdot\frac{du}{dx}$

Quotient Rule: If $y = \frac{u}{v}$ then $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$

Chain Rule: If $y = g(u)$, where $u = f(x)$ then $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$

EXPONENTIAL & LOGARITHMIC FUNCTIONS

$\frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$

$\frac{d}{dx}b^u = b^u\cdot\ln b\cdot\frac{du}{dx}$

$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$

$\frac{d}{dx}\log_b u = \frac{1}{u\ln b}\cdot\frac{du}{dx}$

TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}\sin u = \cos u\cdot\frac{du}{dx}$

$\frac{d}{dx}\csc u = -\csc u\cdot\cot u\cdot\frac{du}{dx}$

$\frac{d}{dx}\cos u = -\sin u\cdot\frac{du}{dx}$

$\frac{d}{dx}\sec u = \sec u\cdot\tan u\cdot\frac{du}{dx}$

$\frac{d}{dx}\tan u = \sec^2 u\cdot\frac{du}{dx}$

$\frac{d}{dx}\cot u = -\csc^2 u\cdot\frac{du}{dx}$

BASIC INTEGRATION RULES

$\int k\;dx = kx + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\;\; n \neq -1$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C,\;\; n \neq 1$

$\int \frac{1}{x}dx = \ln x + C,\;\; x > 0$

If $x<0$ *write* $\ln|x|+C$

$\int e^x dx = e^x + C$

$\int \cos x\cdot dx = \sin x + C$

$\int \sin x\cdot dx = -\cos x + C$

$\int \sec^2 x\cdot dx = \tan x + C$

$\int \csc^2 x\cdot dx = -\cot x + C$

$\int \sec x\tan x\cdot dx = \sec x + C$

$\int \csc x\cot x\cdot dx = -\csc x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C\;\;(a \neq 1)$

QUADRATIC FORMULA

If $ax^2+bx+c=0$, then

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \quad a\neq 0$$

COORDINATE GEOMETRY

For two points: (x_1, y_1) and (x_2, y_2):

distance → $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

slope → $m=\dfrac{y_2-y_1}{x_2-x_1}$

Equation of a line: $y=mx+b$

$y-y_1=m(x-x_1)$

LIMITS OF TRIGONOMETRIC FUNCTIONS

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{\cos x-1}{x}=0$$

GEOMETRIC SERIES and LOGARITHMIC FUNCTIONS

$$S_n=\frac{a(r^n-1)}{r-1}, \quad S=\frac{a}{1-r}, \quad r\neq 1$$

$$b^x=a \;\rightarrow\; x=\log_b a$$

$$\log_b a=\frac{\log_c a}{\log_c b}$$

$$\log_b xy=\log_b x+\log_b y$$

$$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$$

$$\log_b x^n=n\log_b x$$

$$b^x=e^{x\ln b}$$

$$\log_b b^x=x=b^{\log_b x}$$

TRIGONOMETRY

Sine Law: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Cosine Law: $a^2=b^2+c^2-2bc\cos A$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

GEOMETRIC FORMULAS

Triangle: $A=\frac{1}{2}bh$

Rectangle: $A=lw$

Circle: $A=\pi r^2$ $\quad C=2\pi r$

Sphere: $V=\frac{4}{3}\pi r^3$

$SA=4\pi r^2$

Cone: $V=\frac{1}{3}\pi r^2 h$

Cylinder: $V=\pi r^2 h$

$SA=2\pi r^2+2\pi rh$

TRIGONOMETRIC IDENTITIES

$$\sin^2\theta+\cos^2\theta=1$$

$$1+\tan^2\theta=\sec^2\theta$$

$$1+\cot^2\theta=\csc^2\theta$$

$$\sin 2\theta=2\sin\theta\cos\theta$$

$$\cos 2\theta=\cos^2\theta-\sin^2\theta$$

$$\cos 2\theta=1-2\sin^2\theta$$

$$\cos 2\theta=2\cos^2\theta-1$$